

---

# Verifying the LLVM

---

Steve Zdancewic

DeepSpec Summer School 2017



# Vminus Operational Semantics

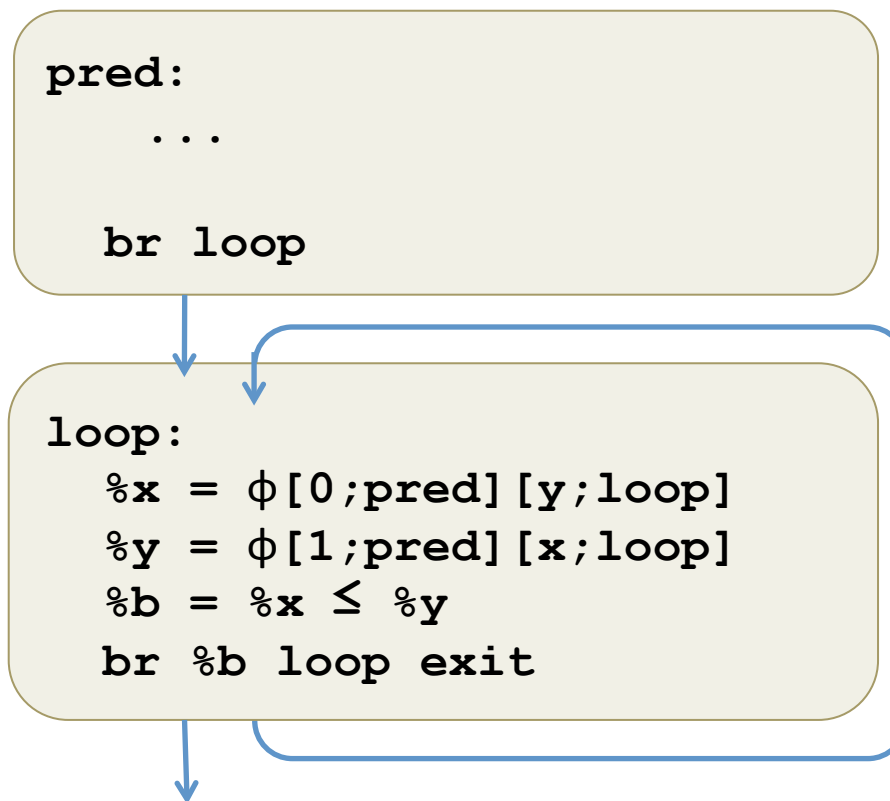
---

- Only 5 kinds of instructions:
  - Binary arithmetic
  - Memory Load
  - Memory Store
  - Terminators
  - Phi nodes
- What is the state of a Vminus program?

# Subtlety of Phi Nodes

---

- Phi-Nodes admit “cyclic” dependencies:



# Semantics of Phi Nodes

---

- The value of the RHS of a phi-defined uid is relative to the state at the entry to the block.
- Option 1:
  - Require all phi nodes to be at the beginning of the block
  - Execute them “atomically, in parallel”
  - (Original Vellvm followed this model)
- Option 2:
  - Keep track of the state upon entry to the block
  - Calculate the RHS of phi nodes relative to the entry state
  - (Vminus follows this model)

---

VminusOpsem.v

# **VMINUS OPERATIONAL SEMANTICS**

# Key SSA Invariant

entry:

`r0 = ...`

`r1 = ...`

`r2 = ...`

`br r0 loop exit`

Definition of  $r_2$ .

loop:

`r3 =  $\phi$ [0;entry][r5;loop]`

`r4 = r1 * r2`

`r5 = r3 + r4`

`r6 = r5 ≥ 100`

`br r6 loop exit`

Uses of  $r_2$ .

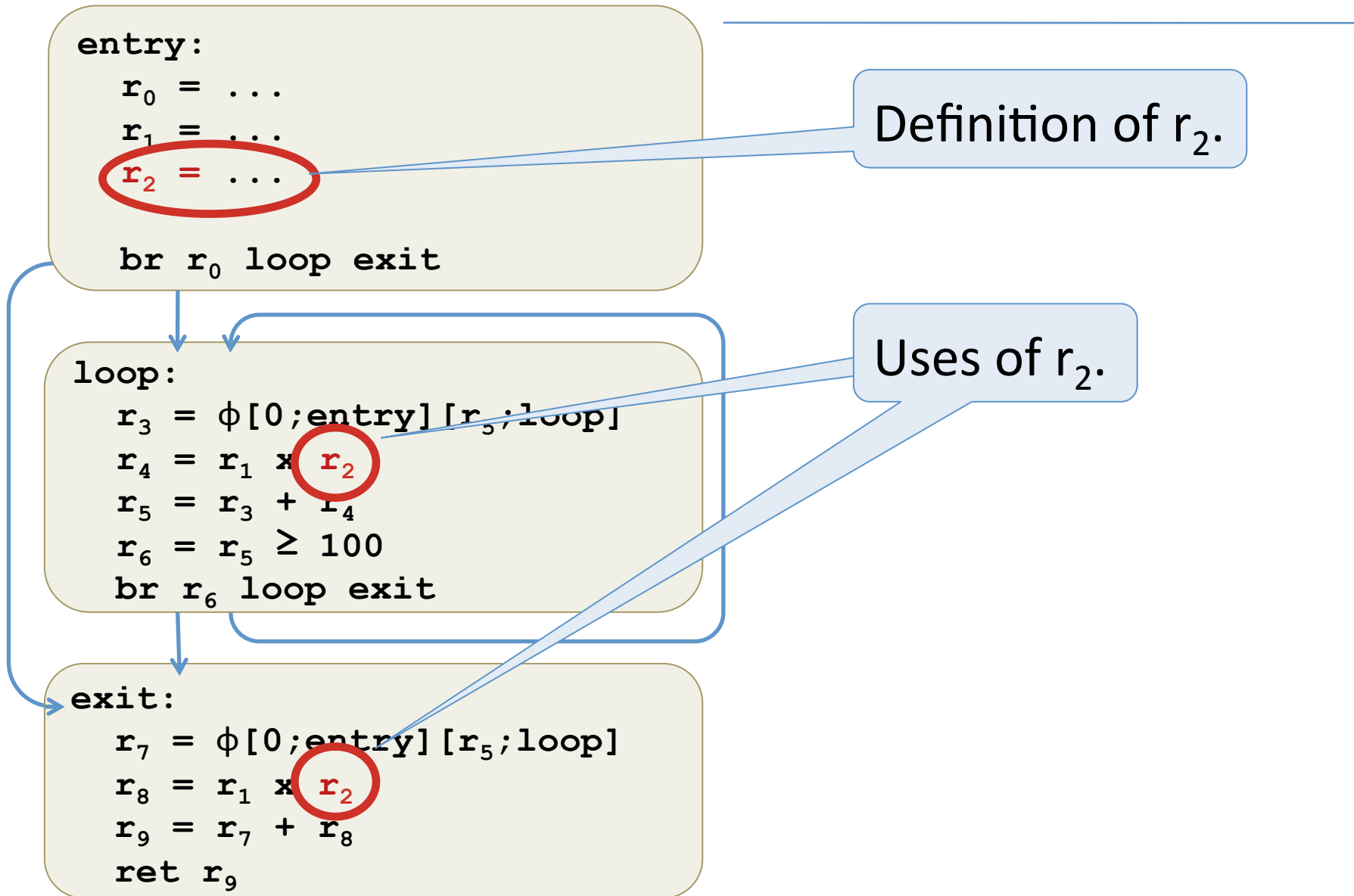
exit:

`r7 =  $\phi$ [0;entry][r5;loop]`

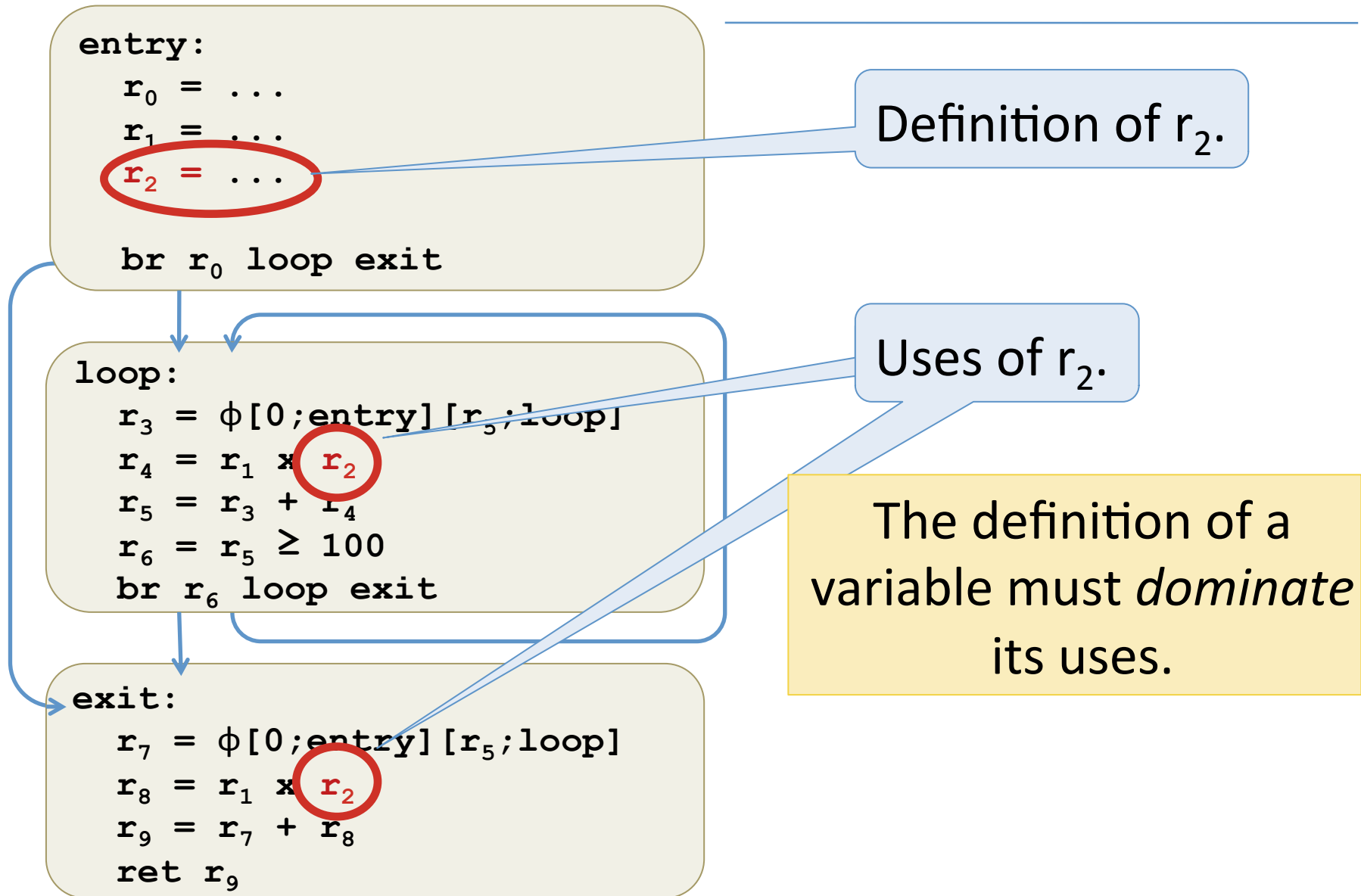
`r8 = r1 * r2`

`r9 = r7 + r8`

`ret r9`



# Key SSA Invariant



# Defining SSA Variable Scope

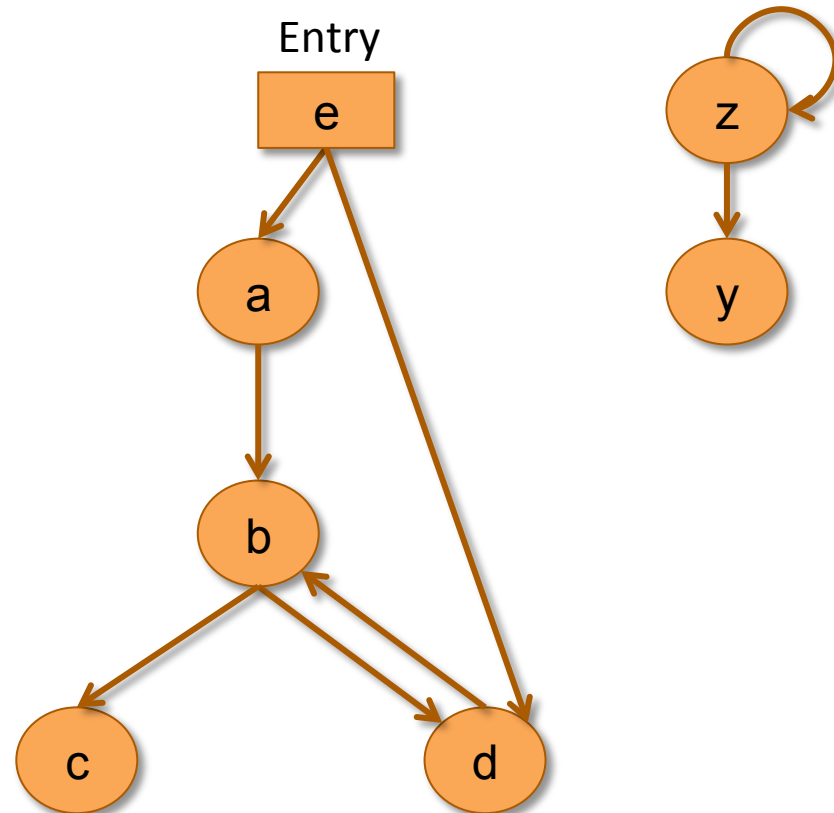
---

*Graph:* g corresponds to a “fine grained” CFG

*Nodes:* program points  
(maybe more than one per block)

*Edges:* “fallthroughs”,  
jump and branch  
instructions

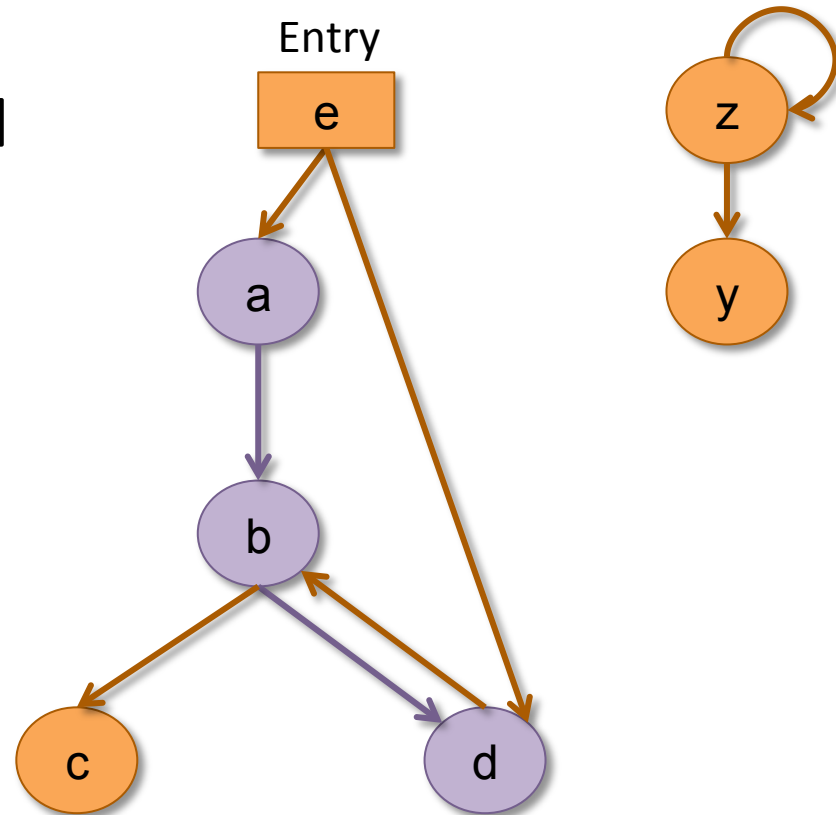
*Distinguished entry*





# Paths

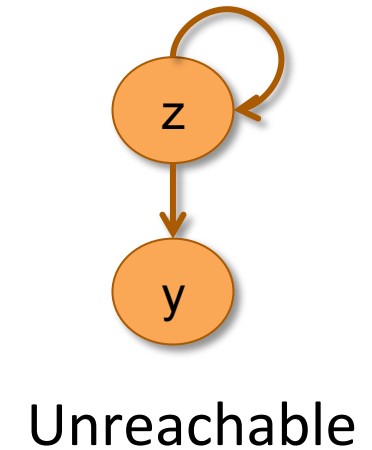
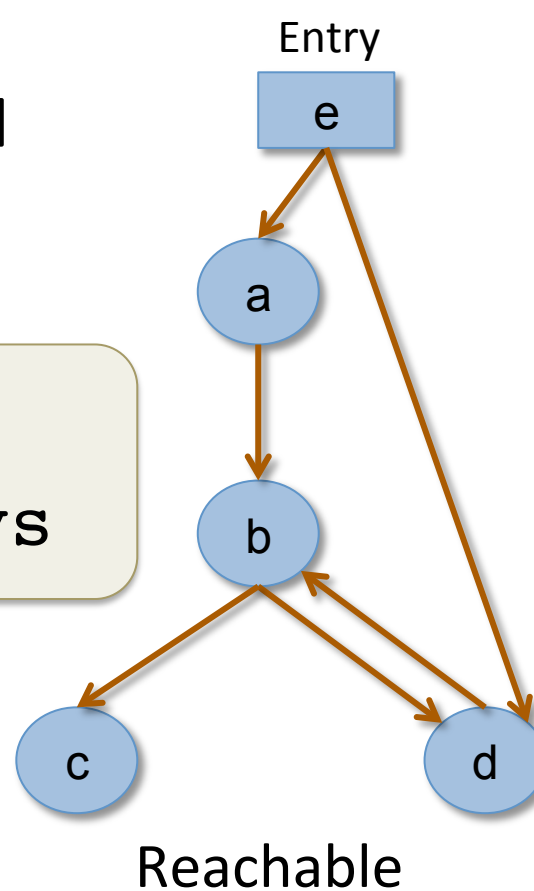
- **Paths:**  
Path g a d [a;b;d]



# Reachability

- Paths:  
Path  $g \ a \ d \ [a;b;d]$
- **Reachability:**  
Reachable  $g \ x$

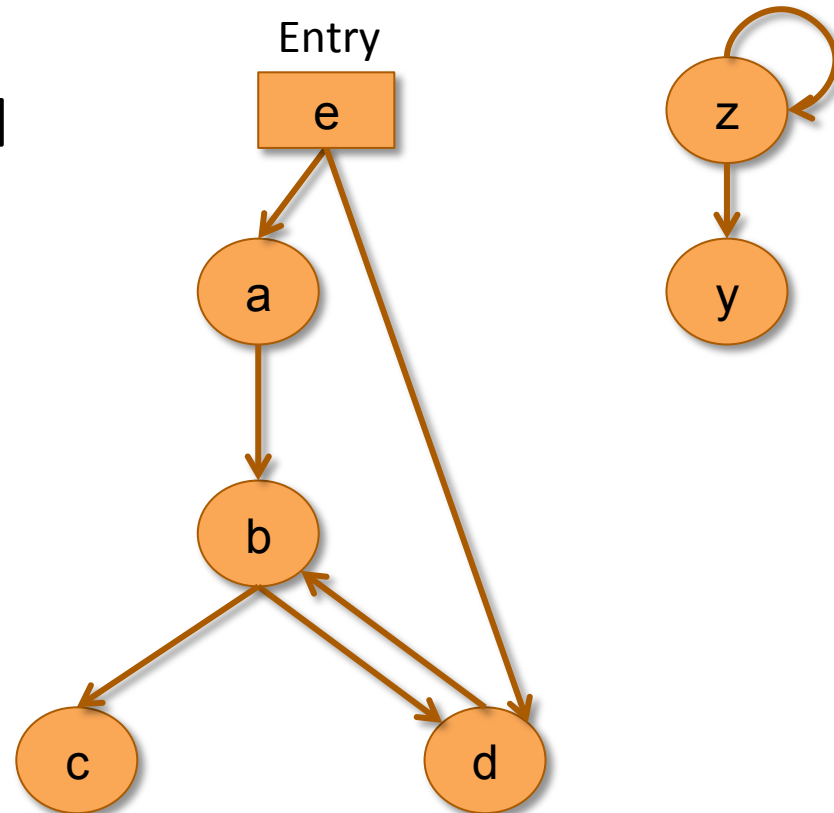
iff  
 $\exists$  vs. Path  $g \ e \ x$  vs



# Domination

- Paths:  
Path  $g$  a d [a;b;d]
- Reachability:  
Reachable  $g$  x
- **Domination:**  
Dom  $g$  b c

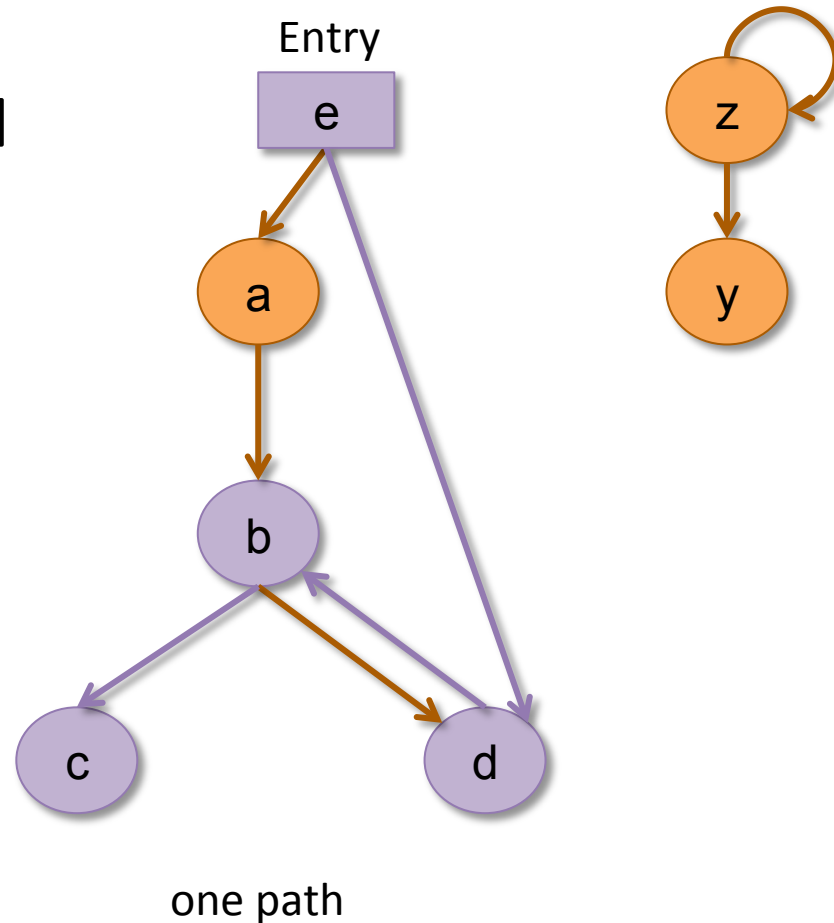
iff every path from  
e to c goes through b.



# Domination

- Paths:  
Path  $g$  a d [a;b;d]
- Reachability:  
Reachable  $g$  x
- **Domination:**  
Dom  $g$  b c

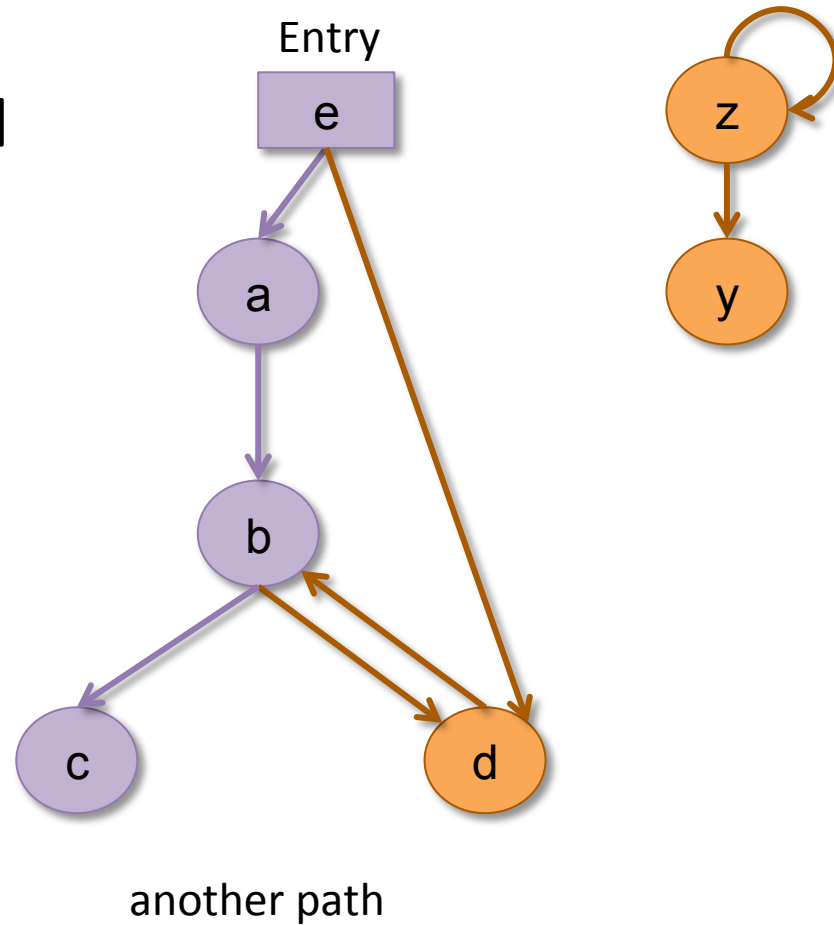
iff every path from  
e to c goes through b.



# Domination

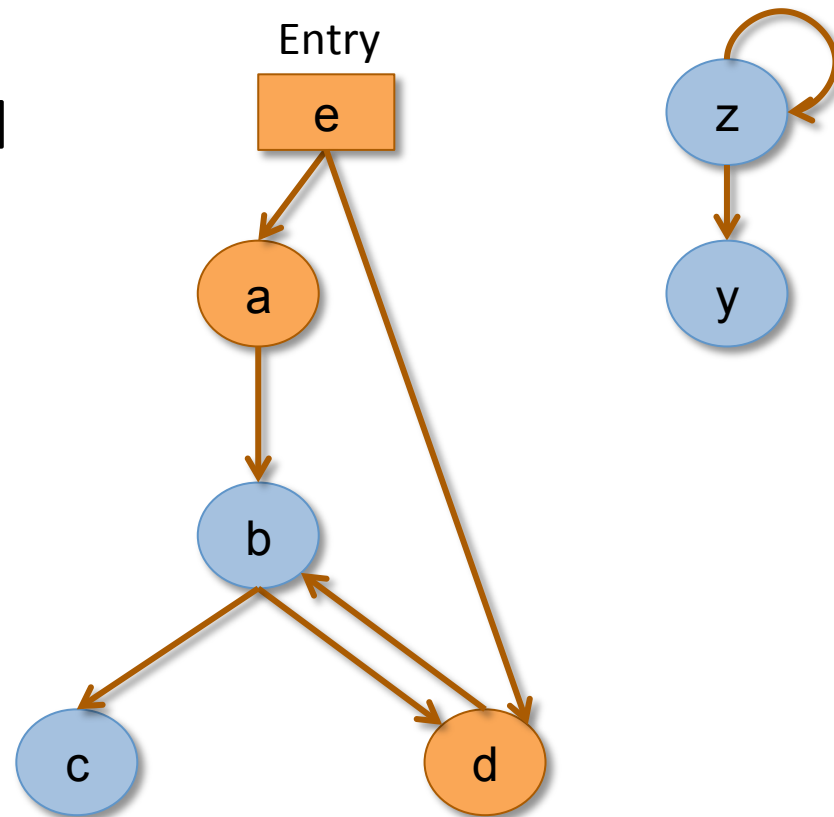
- Paths:  
Path  $g$  a d [a;b;d]
- Reachability:  
Reachable  $g$  x
- **Domination:**  
Dom  $g$  b c

iff every path from  
e to c goes through b.



# Domination

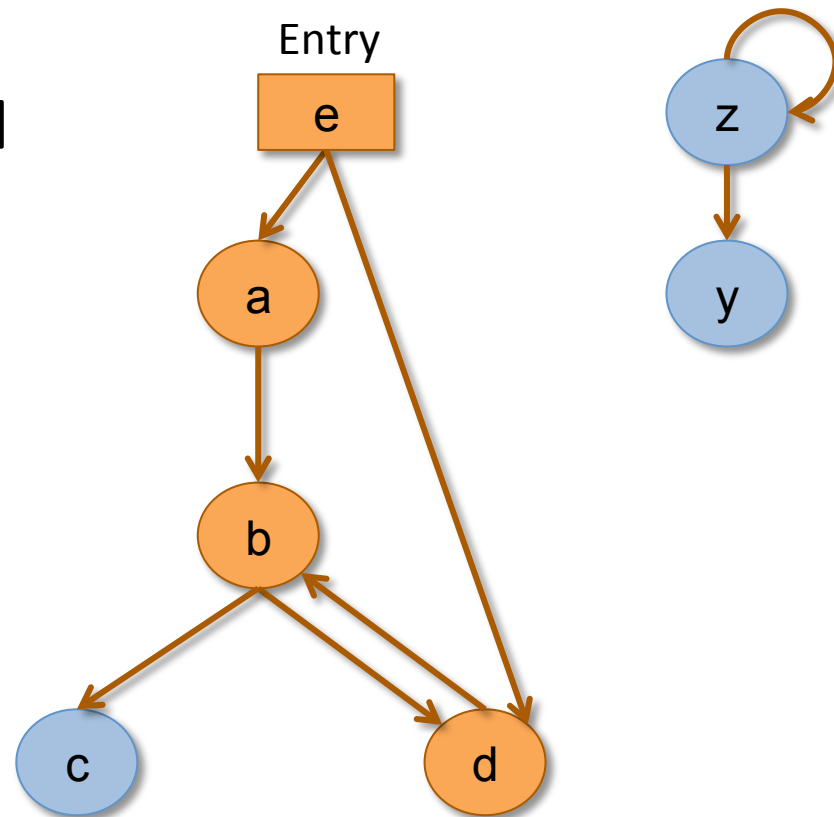
- Paths:  
Path  $g$  a d [a;b;d]
- Reachability:  
Reachable  $g$  x
- **Domination:**  
Dom  $g$  b c



Nodes dominated by b.

# Strict Domination

- Paths:  
Path  $g$  a d [a;b;d]
- Reachability:  
Reachable  $g$  x
- Domination:  
Dom  $g$  b c
- **Strict Domination:**  
SDom  $g$  b c

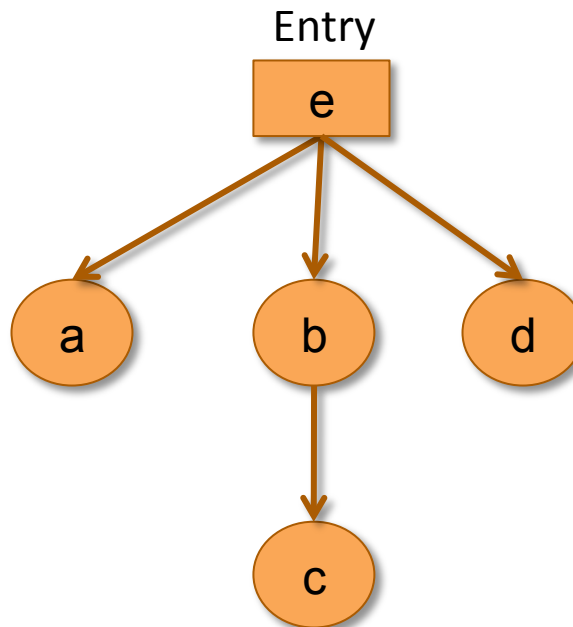


Nodes strictly dominated by b.

# Domination Tree

---

- Order the reachable nodes by (immediate) dominators, you get a tree:

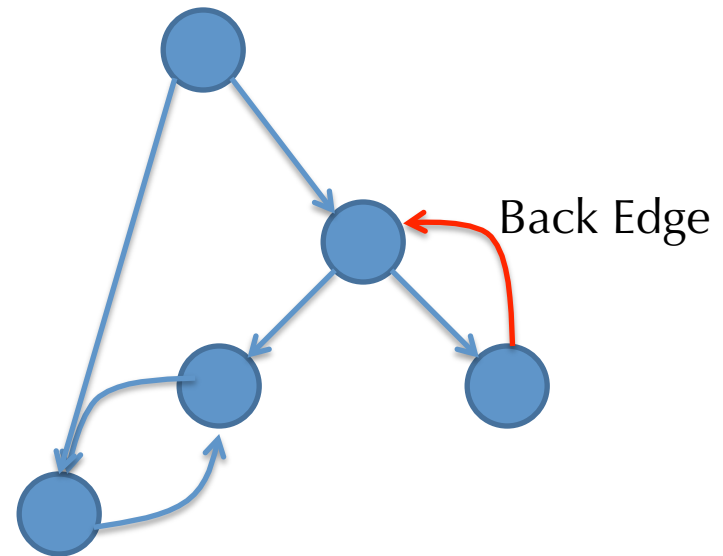




# Control-flow Analysis

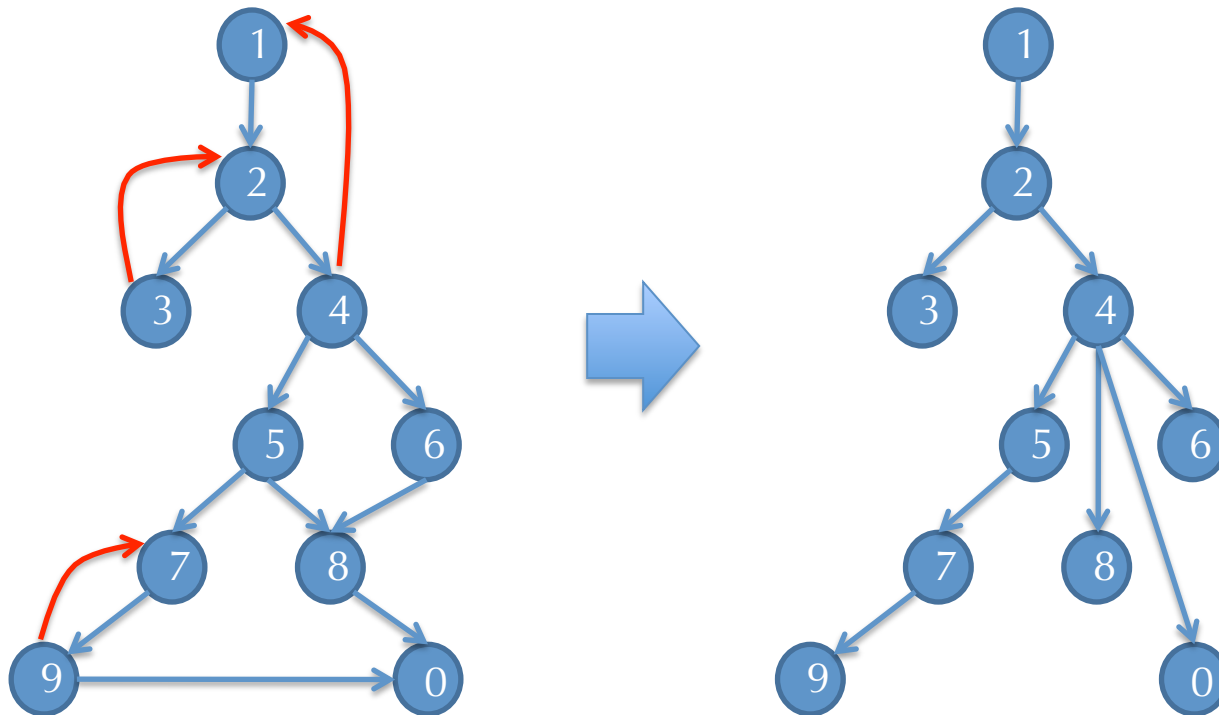
---

- Goal: Identify the loops and nesting structure of the CFG.
- An edge in the graph is a **back edge** if the target node dominates the source node.
- A loop contains at least one back edge.



# Dominator Trees

- Domination is transitive:
  - if A dominates B and B dominates C then A dominates C
- Domination is anti-symmetric:
  - if A dominates B and B dominates A then A = B
- Every flow graph has a dominator tree
  - The Hasse diagram of the dominates relation



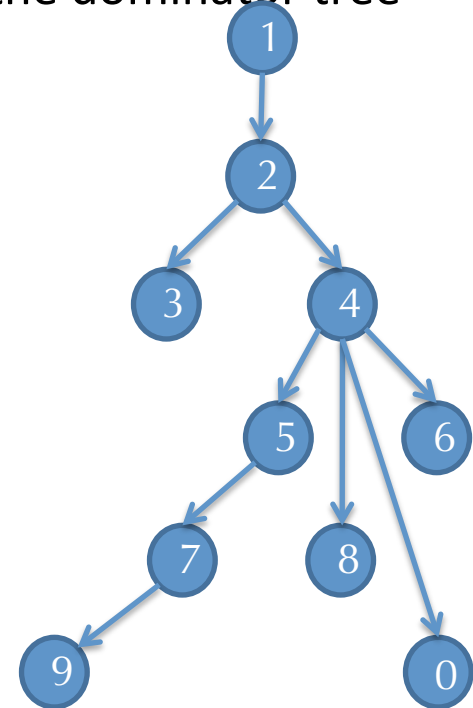
# Dominator Dataflow Analysis

---

- Let  $\text{Dom}[n] = \{m \mid m \text{ dominates } n\}$
- We can define  $\text{Dom}[n]$  as a forward dataflow analysis.
  - Using the framework we saw earlier:  $\text{Dom}[n] = \text{out}[n]$  where:
- “A node B is dominated by another node A if A dominates *all* of the predecessors of B.”
  - $\text{in}[n] := \bigcap_{n' \in \text{pred}[n]} \text{out}[n']$
- “Every node dominates itself.”
  - $\text{out}[n] := \text{in}[n] \cup \{n\}$
- Formally:  $\mathcal{L} = \text{set of nodes ordered by } \subseteq$ 
  - $T = \{\text{all nodes}\}$
  - $F_n(x) = x \cup \{n\}$
  - $\sqcap$  is  $\cap$
- Easy to show monotonicity and that  $F_n$  distributes over meet.
  - Bounded set of variables: so algorithm terminates

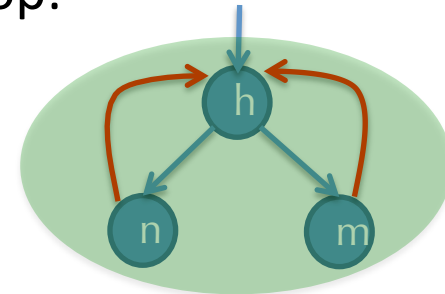
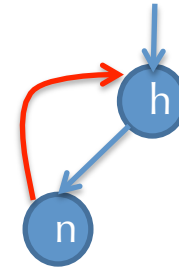
# Improving the Algorithm

- Dom[b] contains just those nodes along the path in the dominator tree from the root to b:
  - e.g. Dom[8] = {1,2,4,8}, Dom[7] = {1,2,4,5,7}
  - There is a lot of sharing among the nodes
- More efficient way to represent Dom sets is to store the dominator *tree*.
  - doms[b] = immediate dominator of b
  - doms[8] = 4, doms[7] = 5
- To compute Dom[b] walk through doms[b]
- Need to efficiently compute intersections of Dom[a] and Dom[b]
  - Traverse up tree, looking for least common ancestor:
  - Dom[8]  $\cap$  Dom[7] = Dom[4]
- See: “A Simple, Fast Dominance Algorithm” Cooper, Harvey, and Kennedy



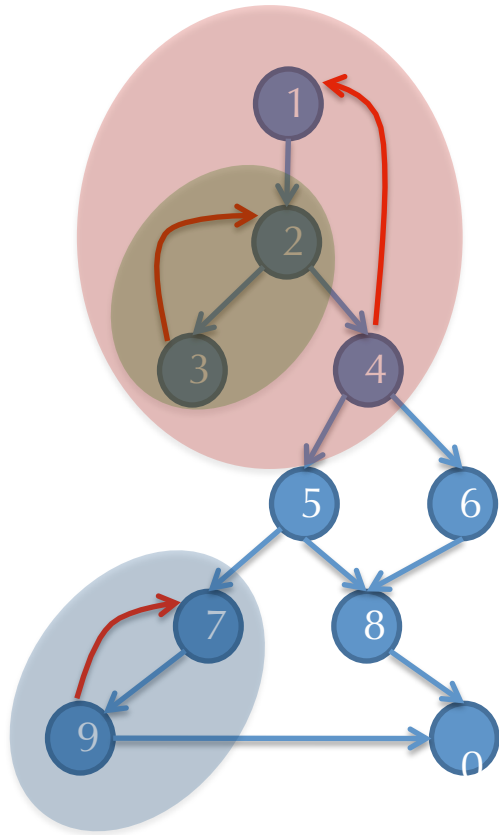
# Completing Control-flow Analysis

- Dominator analysis identifies *back edges*:
  - Edge  $n \rightarrow h$  where  $h$  dominates  $n$
- Each back edge has a *natural loop*:
  - $h$  is the header
  - All nodes reachable from  $h$  that also reach  $n$  without going through  $h$
- For each back edge  $n \rightarrow h$ , find the natural loop:
  - $\{n' \mid n \text{ is reachable from } n' \text{ in } G - \{h\}\} \cup \{h\}$
- Two loops may share the same header: merge them
- Nesting structure of loops is determined by set inclusion
  - Can be used to build the control tree



# Example Natural Loops

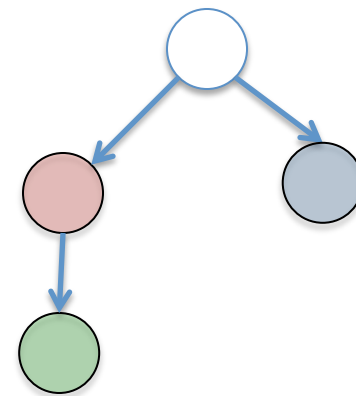
---



Natural Loops



Control Tree:



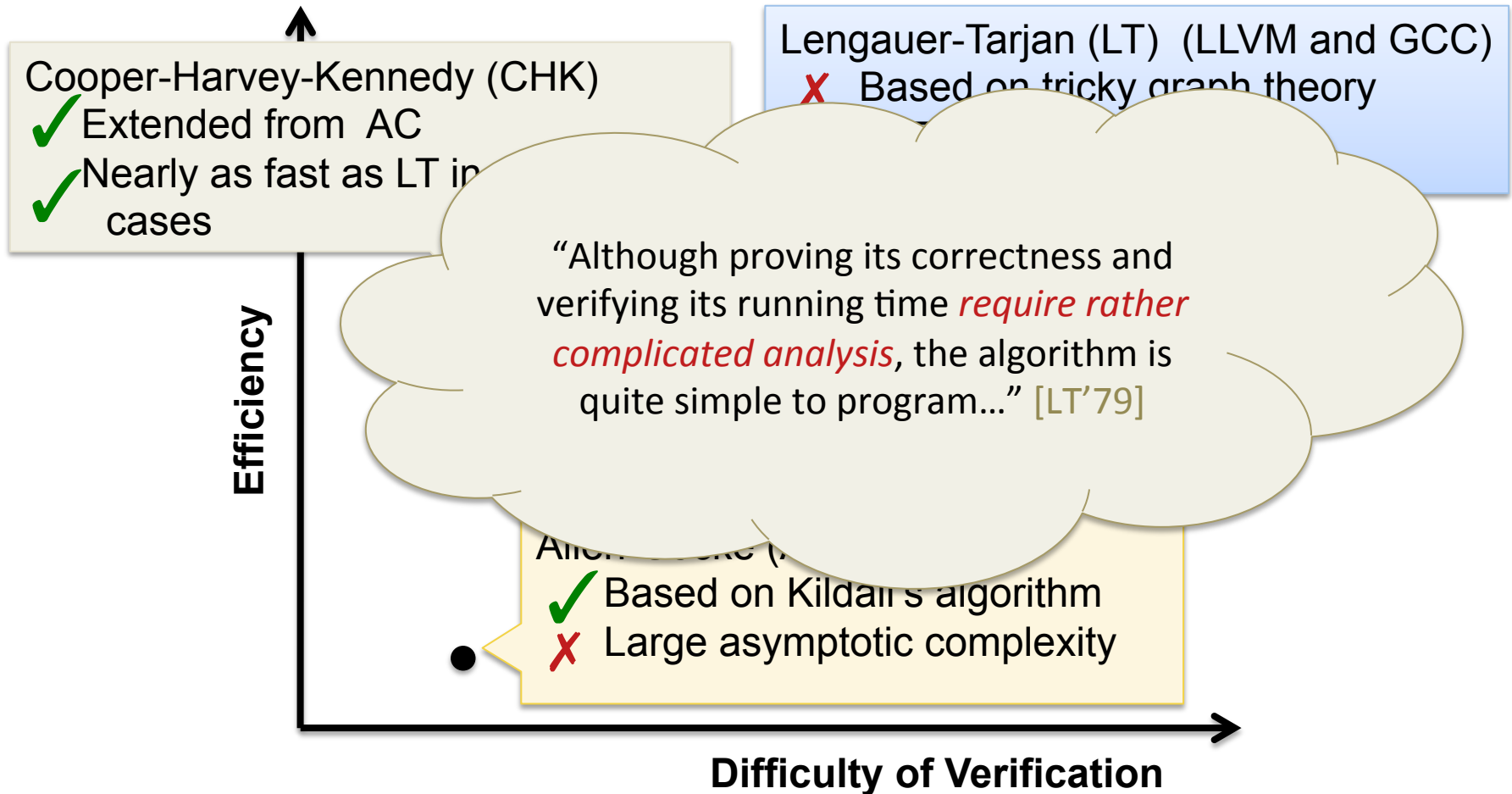
The control tree depicts the nesting structure of the program.

# Uses of Control-flow Information

---

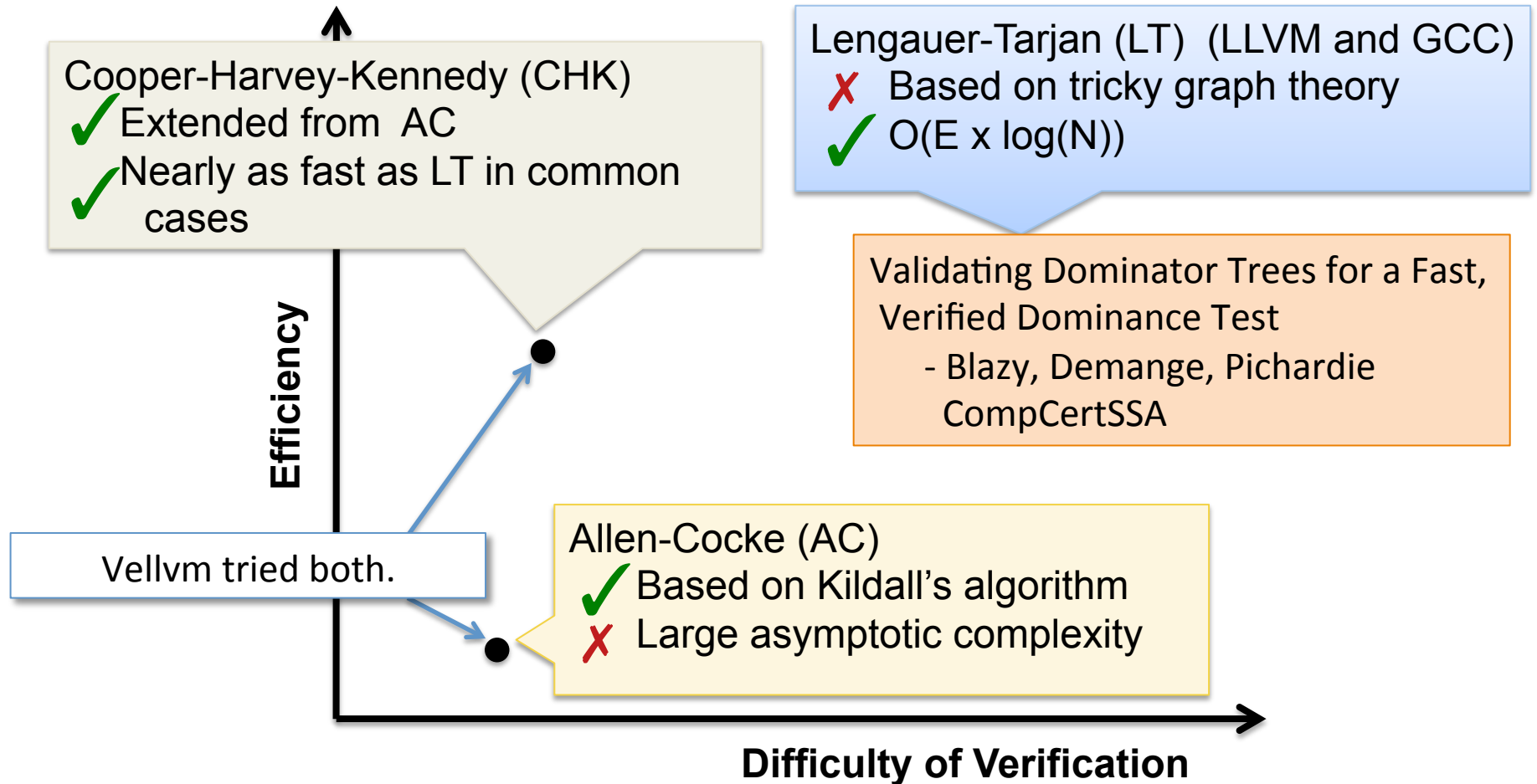
- Loop nesting depth plays an important role in optimization heuristics.
  - Deeply nested loops pay off the most for optimization.
- Need to know loop headers / back edges for doing
  - loop invariant code motion
  - loop unrolling

# Dominator Algorithm Tradeoffs





# Dominator Algorithm Tradeoffs



---

Dom.v

Kildall.v

DomKildall.v

# **DOMINATORS**

# Safety Properties

---

- A well-formed program never accesses undefined variables.

If  $\vdash f$  and  $f \vdash \sigma_0 \mapsto^* \sigma$  then  $\sigma$  is not stuck.

$\vdash f$                       program  $f$  is well formed

$\sigma$                          program state

$f \vdash \sigma \mapsto^* \sigma$  evaluation of  $f$

- *Initialization:*

If  $\vdash f$  then  $\text{wf}(f, \sigma_0)$ .

- *Preservation:*

If  $\vdash f$  and  $f \vdash \sigma \mapsto \sigma'$  and  $\text{wf}(f, \sigma)$  then  $\text{wf}(f, \sigma')$

- *Progress:*

If  $\vdash f$  and  $\text{wf}(f, \sigma)$  then  $f \vdash \sigma \mapsto \sigma'$

# Safety Properties

- A well-formed program never accesses undefined variables.

If  $\vdash f$  and  $f \vdash \sigma_0 \mapsto^* \sigma$  then  $\sigma$  is not stuck.

$\vdash f$  program  $f$  is well formed

$\sigma$  program state

$f \vdash \sigma \mapsto^* \sigma$  evaluation of  $f$

- *Initialization:*

If  $\vdash f$  then  $wf(f, \sigma_0)$ .

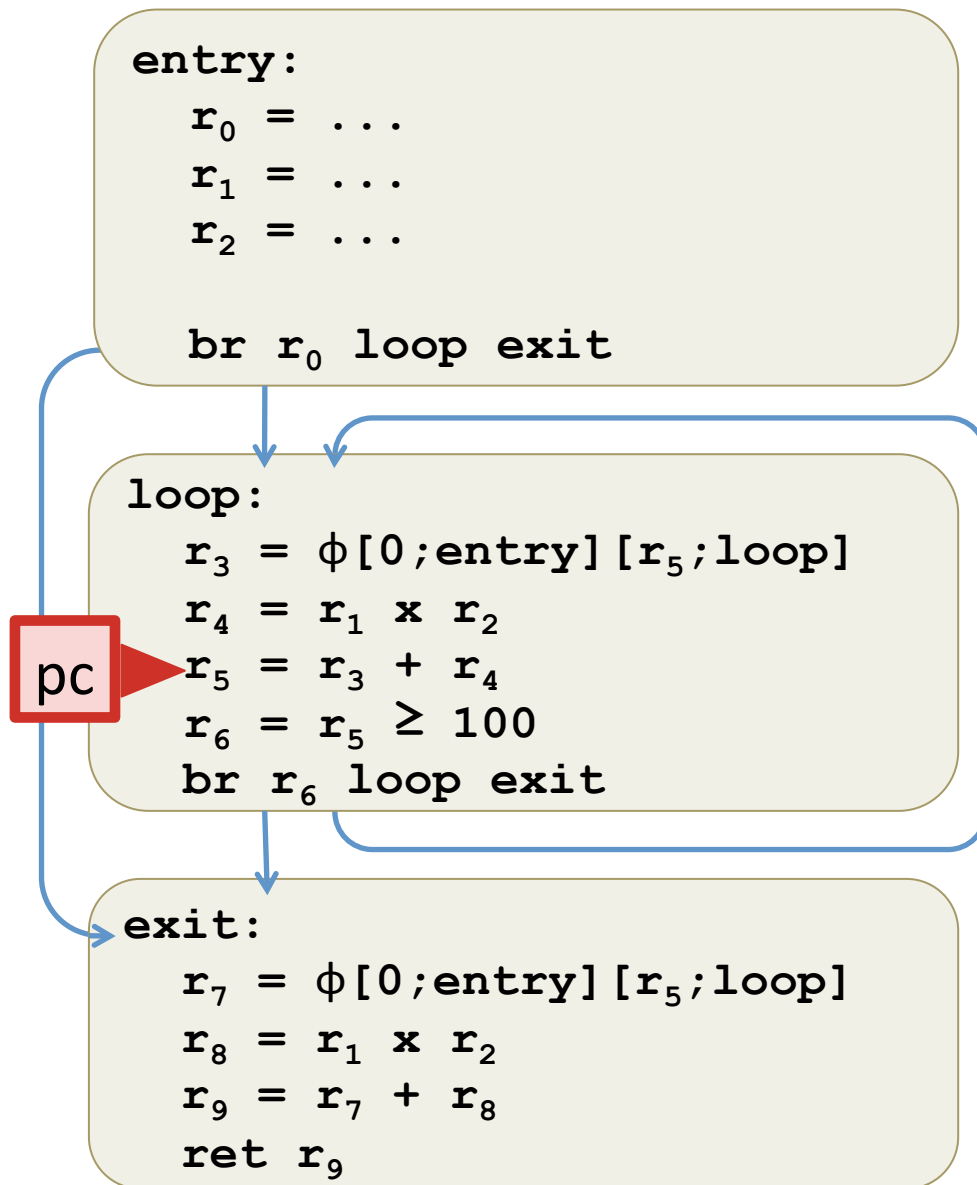
- *Preservation:*

If  $\vdash f$  and  $f \vdash \sigma \mapsto \sigma'$  and  $wf(f, \sigma)$  then  $wf(f, \sigma')$

- *Progress:*

If  $\vdash f$  and  $wf(f, \sigma)$  then  $done(f, \sigma)$  or  $stuck(f, \sigma)$  or  $f \vdash \sigma \mapsto \sigma'$

# Well-formed States

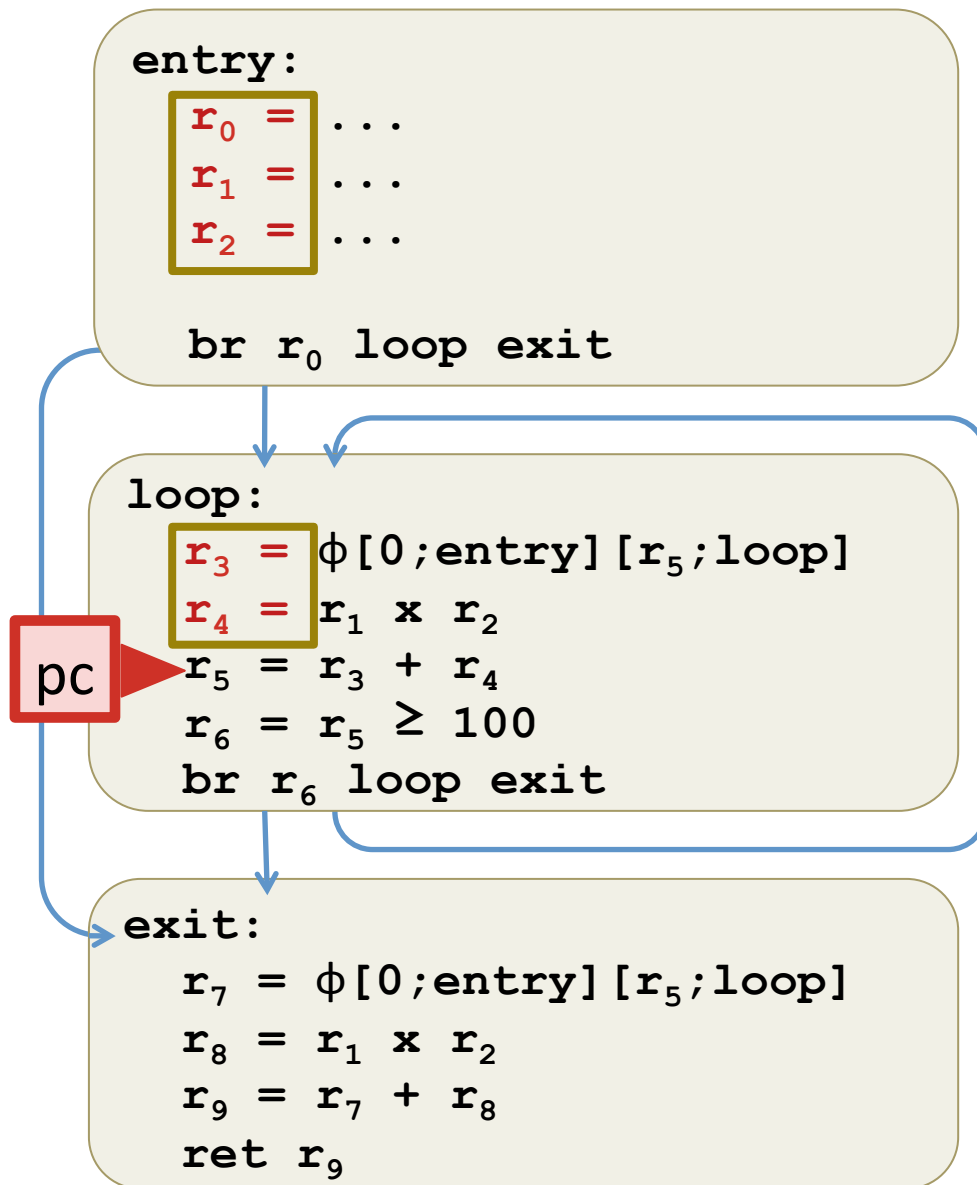


State  $\sigma$  is:

pc = program counter

$\delta$  = local values

# Well-formed States (Roughly)



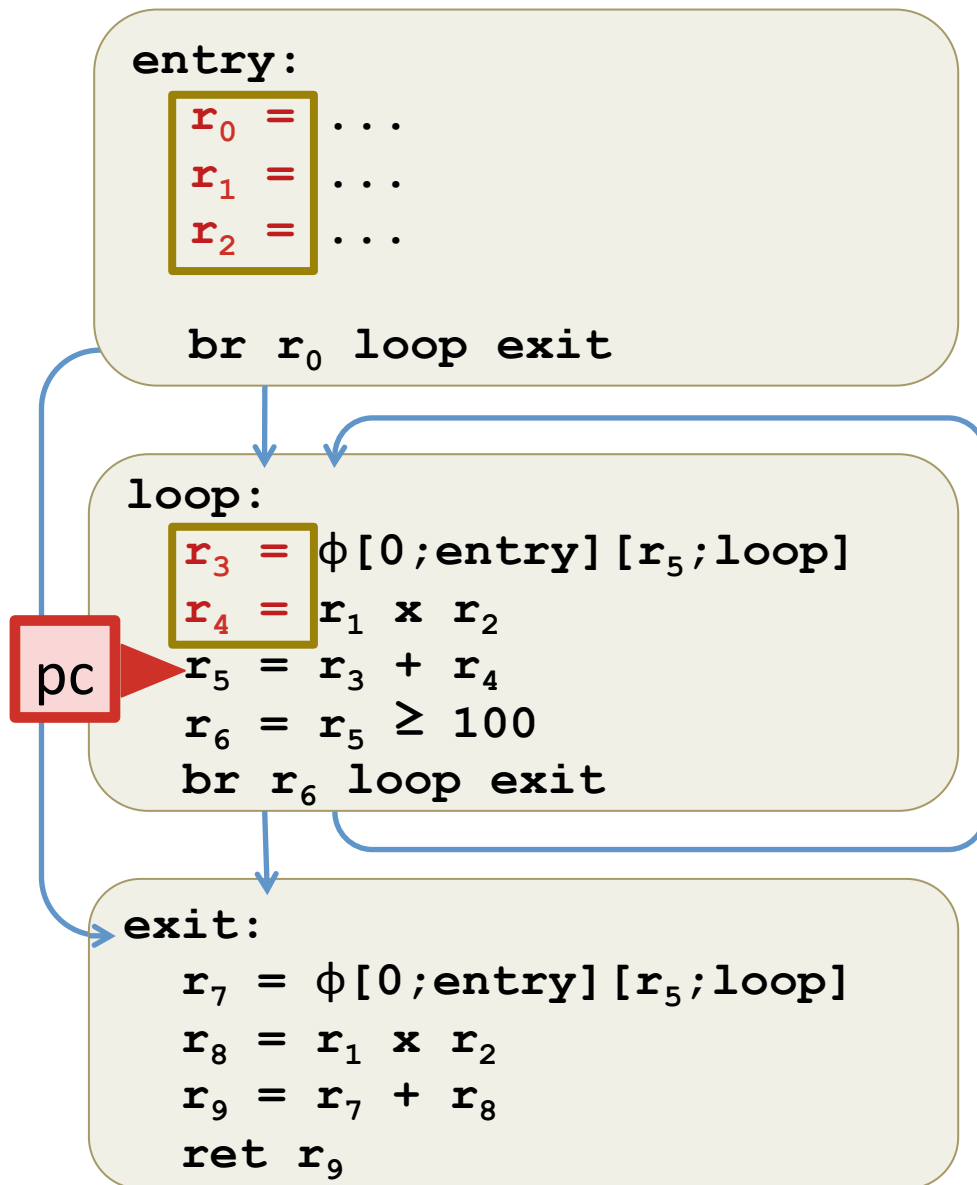
State  $\sigma$  is:

pc = program counter

$\delta$  = local values

$\text{sdom}(f, \text{pc})$  = variable defns.  
that *strictly dominate* pc.

# Well-formed States (Roughly)



State  $\sigma$  contains:

pc = program counter

$\delta$  = local values

mem = memory

$\text{sdom}(f, \text{pc})$  = variable defns.  
that *strictly dominate* pc.

$\text{wf}(g, \sigma) =$

$\forall r \in \text{sdom}(f, \text{pc}). \exists v. \delta(r) = \lfloor v \rfloor$

“All variables in scope  
are initialized.”

---

VminusStatics.v

# **VMINUS STATIC SEMANTICS**